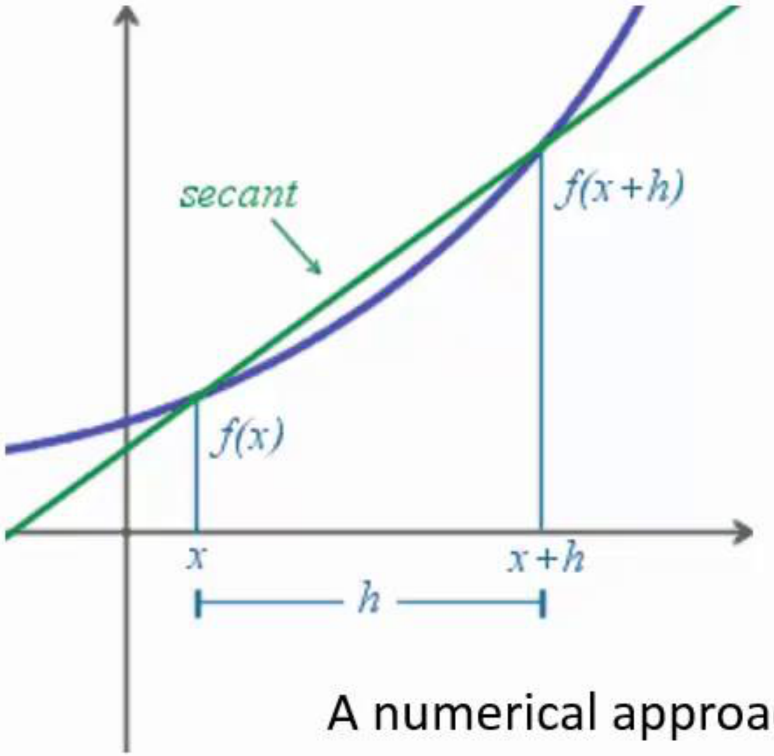
Numerical Differentiation



The derivative of a function $y = f(x)$ is a measure of how y changes with x.

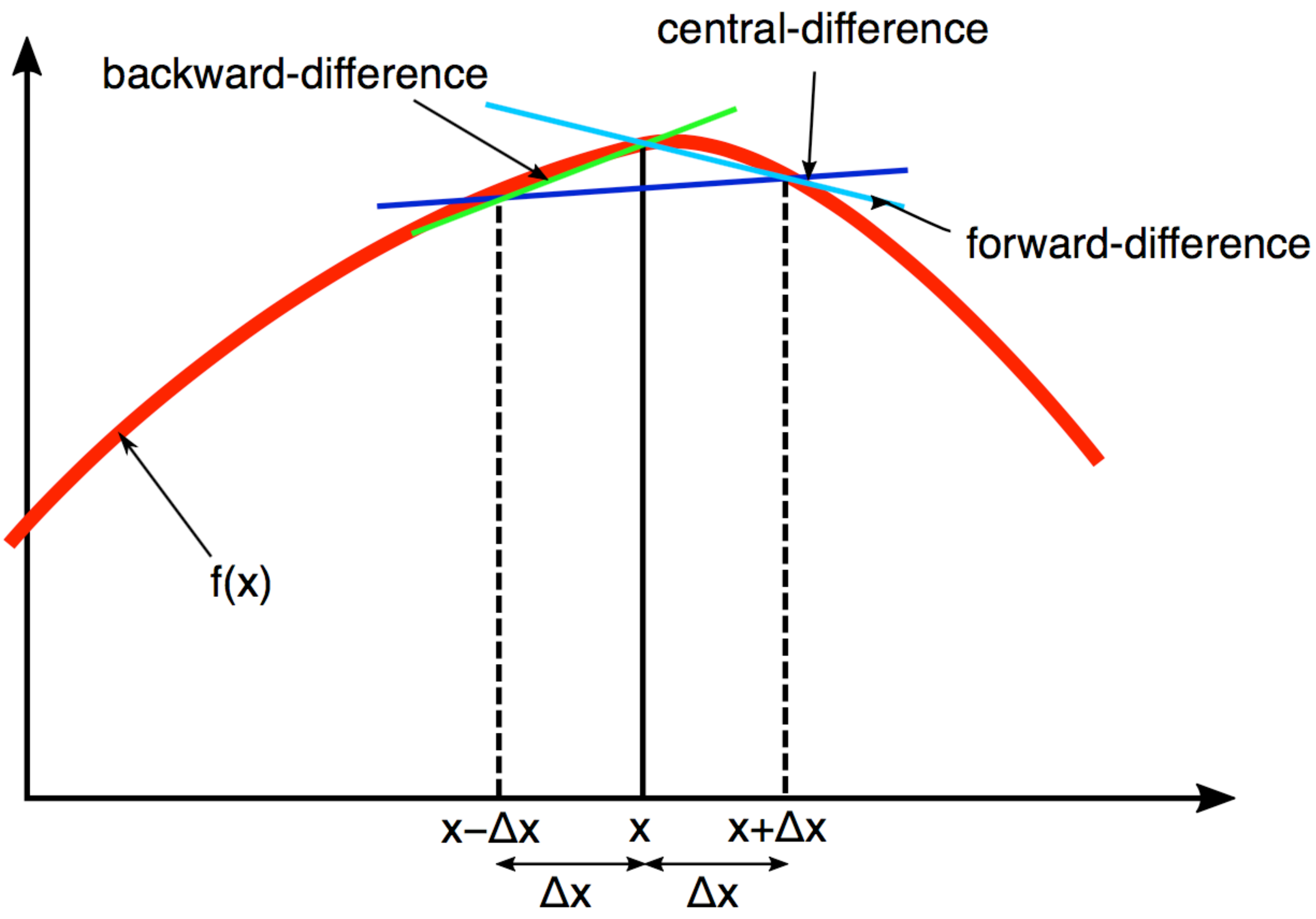
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

A numerical approach to the derivative of a function $y = f(x)$ is:

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note! We will use MATLAB in order to find the numeric solution – not the analytic solution

Image source: https://www.halvorsen.blog


central-difference
backward-difference
forward-difference
f(x)
x–Δx
x
x+Δx
Δx
Δx

Derivative at x_0	**Error**	**Difference Type**
$\frac{dy}{dx} = \frac{y_1 - y_0}{h}$	O(h)	Forward
$\frac{dy}{dx} = \frac{y_0 - y_{-1}}{h}$	O(h)	Backward
$\frac{dy}{dx} = \frac{y_1 - y_{-1}}{2h}$	$O(h^2)$	Central best
$\frac{d^2y}{dx^2} = \frac{y_2 - 2y_1 + y_0}{h^2}$	O(h)	Forward
$\frac{d^2y}{dx^2} = \frac{y_0 - 2y_{-1} + y_{-2}}{h^2}$	O(h)	Backward
$\frac{d^2y}{dx^2} = \frac{y_1 - 2y_0 + y_{-1}}{h^2}$	$O(h^2)$	Central
$\frac{d^3y}{dx^3} = \frac{y_3 - 3y_2 + 3y_1 - y_0}{h^3}$	O(h)	Forward
$\frac{d^3y}{dx^3} = \frac{y_0 - 3y_{-1} + 3y_{-2} - y_{-3}}{h^3}$	O(h)	Backward
$\frac{d^3y}{dx^3} = \frac{y_2 - 2y_1 + 2y_{-1} - y_{-2}}{2h^3}$	$O(h^2)$	Central