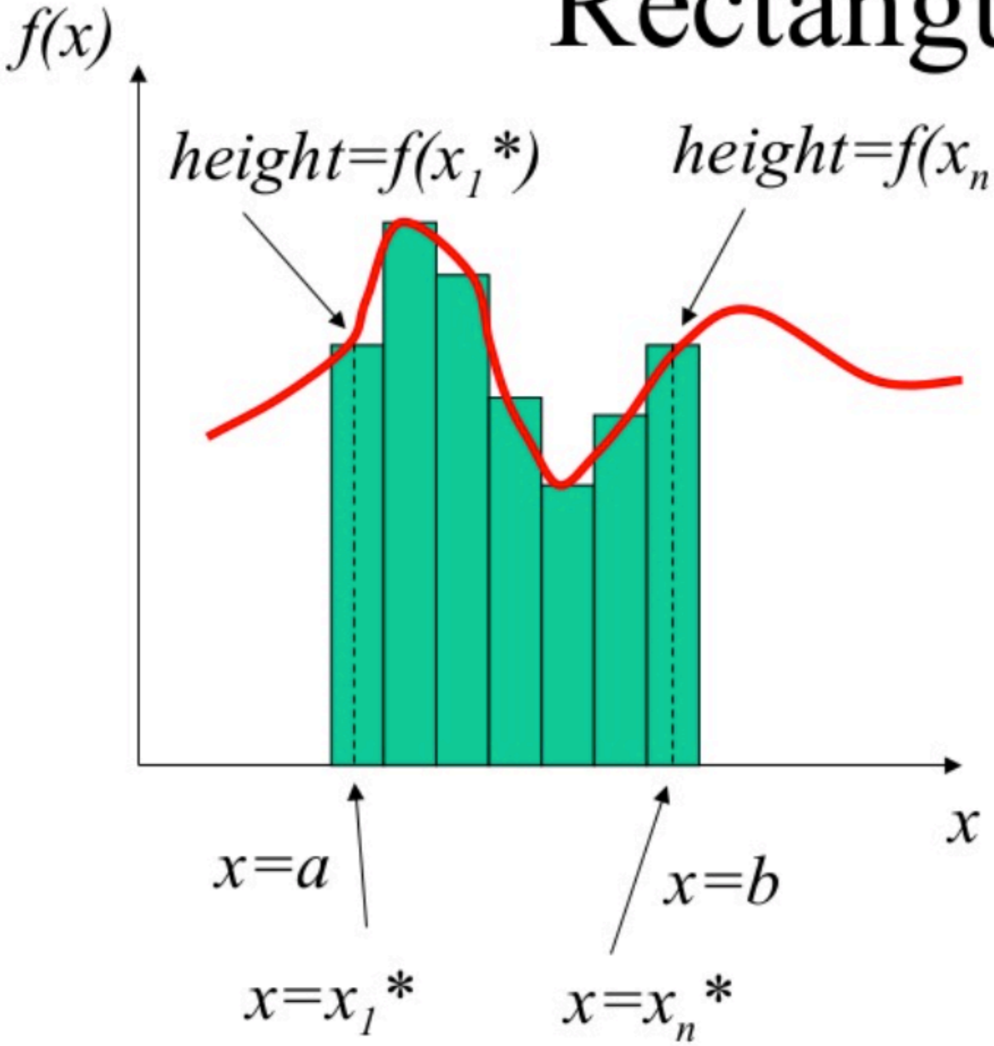
Rectangular Rule



Approximate the integration, $\int_a^b f(x)dx$, that is the area under the curve by a series of rectangles as shown.

The base of each of these rectangles is $\Delta x=(b-a)/n$ and its height can be expressed as $f(x_i^*)$ where x_i^* is the midpoint of each rectangle

$$\int_a^b f(x)dx = f(x_1^*)\Delta x + f(x_2^*)\Delta x + .. f(x_n^*)\Delta x$$

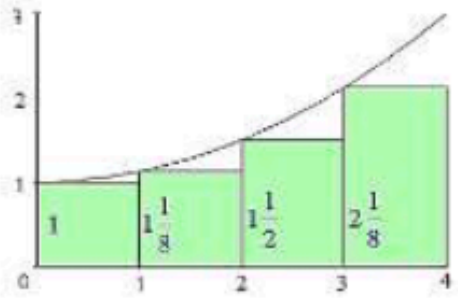
$$= \Delta x[f(x_1^*) + f(x_2^*) + .. f(x_n^*)]$$

Left-hand Rectangular Approximation Method (LRAM / L_n)

We could estimate the area under the curve by drawing rectangles touching at their left corners.

Approximate area:

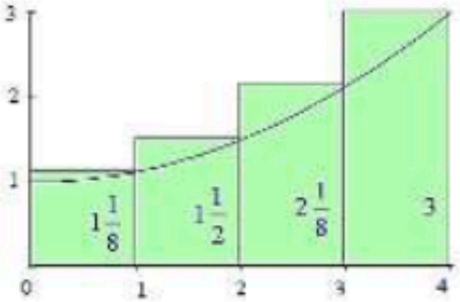
$$L_4 = 1 + 1\frac{1}{8} + 1\frac{1}{2} + 2\frac{1}{8} = 5\frac{3}{4} = 5.75$$



Right-hand Rectangular Approximation Method (RRAM / R_n)

Approximate area:

$$R_4 = 1\frac{1}{8} + 1\frac{1}{2} + 2\frac{1}{8} + 3 = 7\frac{3}{4} = 7.75$$



Midpoint Rectangular Approximation Method (MRAM / M_n)

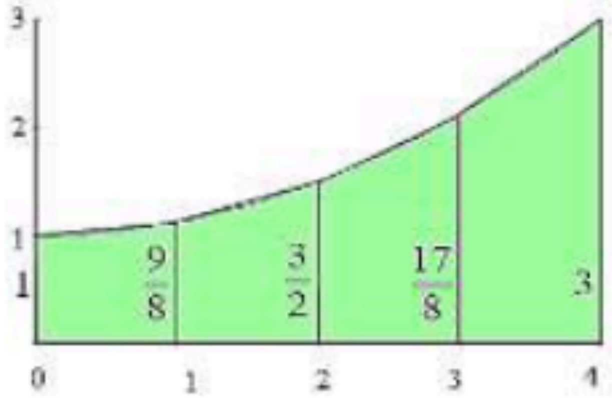
Approximate area:

$M_4 = 6.625$

Trapezoidal Approximation Method (T_n)

Approximate area:

$$T=\frac{1}{2}\left(1+\frac{9}{8}\right)+\frac{1}{2}\left(\frac{9}{8}+\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{2}+\frac{17}{8}\right)+\frac{1}{2}\left(\frac{17}{8}+3\right)$$



$T_4 = 6.75$

Eexact value for area under the curve $y=\frac{1}{8}x^2+1$ $\quad 0\leq x\leq 4$

$$A=\int_0^4 \frac{1}{8}x^2+1\,dx=\frac{1}{24}x^3+x\Big|_0^4=6.\bar{6}$$

$L_4 = 5.75$
$R_4 = 7.75$
$M_4 = 6.625$
$M_8 = 6.65624$
$T_4 = 6.75$